

Volumetric Flow Rate Measurement

Gerald Recktenwald
Portland State University
Department of Mechanical Engineering
gerry@me.pdx.edu

These slides are a supplement to the lectures in ME 449/549 *Thermal Management Measurements* and are © 2006, Gerald W. Recktenwald, all rights reserved. The material is provided to enhance the learning of students in the course, and should only be used for educational purposes. The material in these slides is subject to change without notice.

The PDF version of these slides may be downloaded or stored or printed only for noncommercial, educational use. The repackaging or sale of these slides in any form, without written consent of the author, is prohibited.

The latest version of this PDF file, along with other supplemental material for the class, can be found at www.me.pdx.edu/~gerry/class/ME449. Note that the location (URL) for this web site may change.

Version 0.75 April 20, 2006

Overview

1. Motivation
 - System curves and loss coefficients
 - Fan curves
 - Control of flow for heat transfer experiments
2. Methods of flow rate measurement
3. Flow bench
 - Primary components and principle of operation
 - Fan curve measurement
 - Loss coefficient measurement

Steady Flow Energy Equation

Steady flow energy equation

$$\left[\frac{p}{\rho g} + \frac{V^2}{2g} + z \right]_1 = \left[\frac{p}{\rho g} + \frac{V^2}{2g} + z \right]_2 + h_{\text{loss}} - h_{\text{fan}} \quad (1)$$

where

- p is the static pressure,
- ρ is the fluid density,
- V is the average velocity at a cross-section,
- z is the elevation relative to a datum,
- h_{loss} is the head loss due to friction, kinetic energy dissipation, etc.,
- h_{fan} is head gain from a fan (pump)

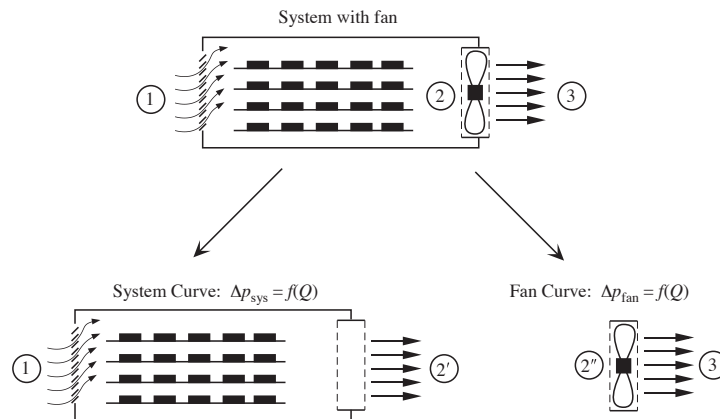
Steady Flow Energy Equation

For air-cooled systems, gravitational effects are negligible, so the energy equation can be rewritten as

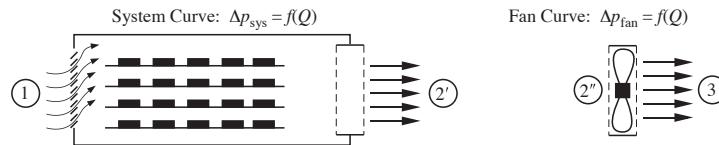
$$\frac{p_1 - p_2}{\rho g} + \frac{V_1^2 - V_2^2}{2g} = h_{\text{loss}} - h_{\text{fan}} \quad (2)$$

System Curve (1)

To define the system curve for an electronics enclosure, we need to separate the fan from the system.



System Curve (2)



On the left, station 2' is the exit pressure necessary to draw the a given flow rate through the system when the fan is not powered. On the right, station 2'' is the upstream condition for the fan during a fan test, i.e., when the fan is not connected to a system.

System Curve (3)

Apply energy equation between stations 1 and 3:

$$p_1 = p_3 \implies h_{\text{loss}} = h_{\text{fan}}$$

In words: The head input by the fan is matched by the overall head loss for the system.

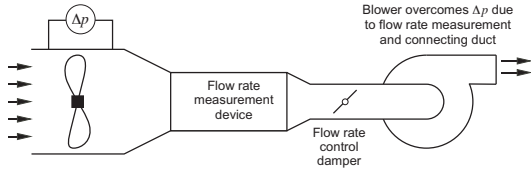
Apply energy equation between stations 1 and 2'

$$\frac{p_1 - p_{2'}}{\rho g} = h_{\text{sys}} \quad (3)$$

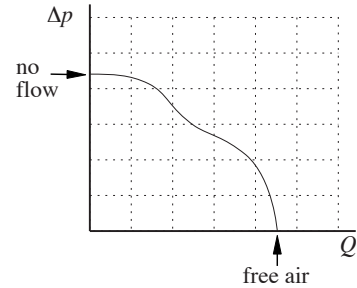
This *defines* the head loss for the system

Fan Curve (1)

Conceptual measurement apparatus:



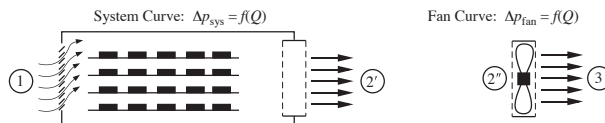
Typical result (a fan curve):



Δp is the *pressure rise* across the fan.

Q is the volumetric flow rate through the fan

Fan Curve (2)



Apply steady flow energy equation between stations 2'' and 3:

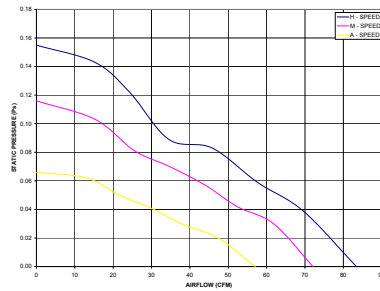
$$p_3 - p_{2''} = \rho g h_{\text{fan}}$$

The pressure *rise* across the fan corresponds to a gain in head. The head gain is matched by head losses elsewhere in the system.

Fan Curve (3)

For vendor-specific information see (as of April 2006)

http://www.comairrotron.com/engineering_notes.asp



Methods of Flow Rate Measurement

Measurement of fan curves and system curves requires

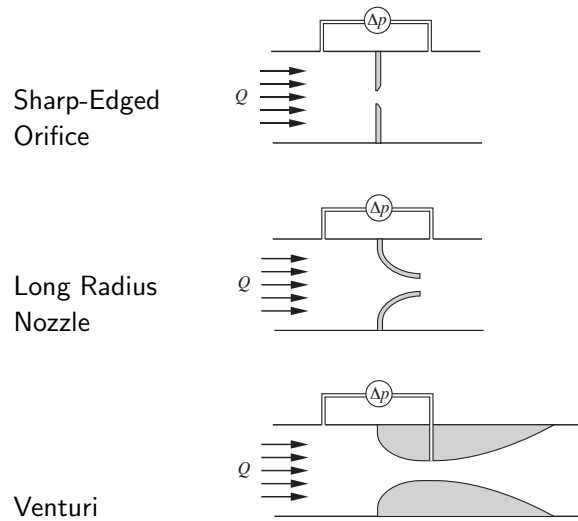
- Control of the flow rate
- Measurement of the flow rate
- Measurement of Δp across the device under test (DUT)

Measuring the Δp across the DUT is relatively simple.

Methods of flow rate measurement

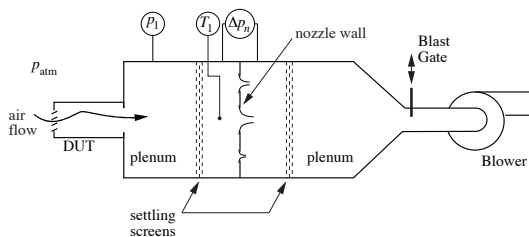
- Velocity profile measurement
- Laminar flow meters
- Rotameters
- Turbine flow meters
- Obstruction flow meters

Obstruction Flow Meters



Flow Bench

A flow bench is a device for providing a controlled and measurable flow rate to or from a device under test (DUT).

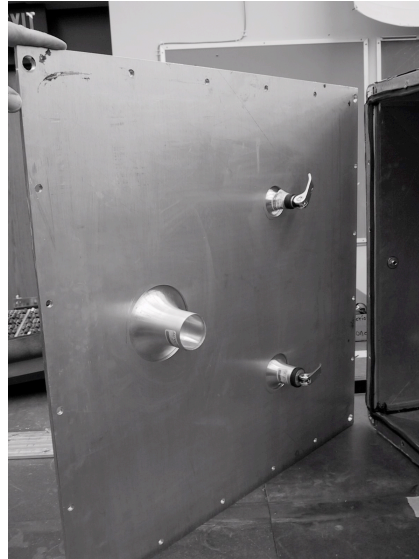


- Two plenums
- Nozzle wall
- Flow control damper (blast gate)
- Blower
- Instrumentation

Nozzle Wall

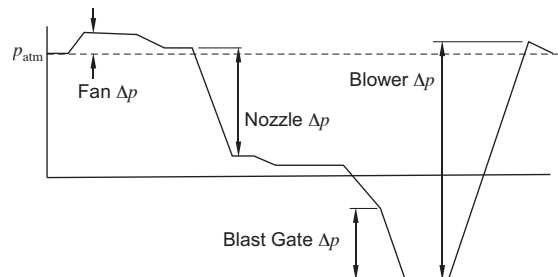
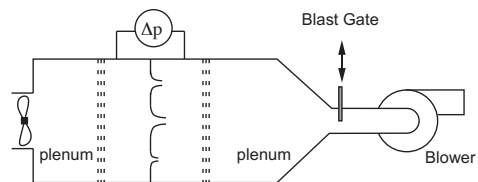
Three nozzles are mounted in an aluminum sheet that separates the two plenums.

Using custom made rubber stoppers, the nozzles can be operated one at a time, or in parallel combinations.



Role of the Blower

- Blower overcomes Δp due to pressure losses
- Largest pressure drops are due to nozzles and blast gate



Control of Flow Rate

During fan curve measurement

- Blast gate controls system flow rate.
- Flow rate determines fan flow rate.
- At a given flow rate, the fan produces a fixed pressure rise.

During system curve measurement

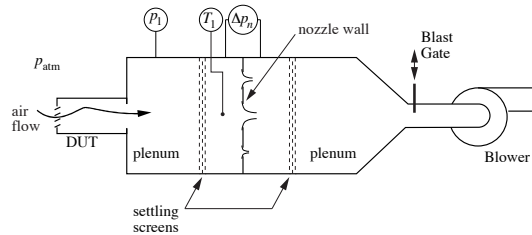
- Blast gate controls system flow rate.
- Flow rate determines pressure drop through DUT.

System Curve or Loss Coefficient Measurement

Locate the Device Under Test (DUT) at inlet of flow bench

System curve:

$$\Delta p_{\text{sys}} = p_{\text{atm}} - p_1 = f(Q) \quad (4)$$



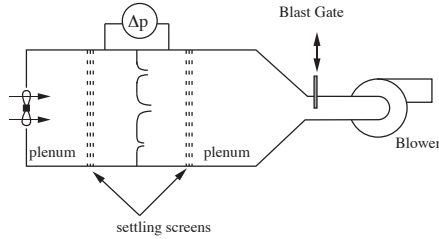
Δp_{sys} is pressure drop necessary to overcome flow resistance of DUT,

p_{atm} is local ambient pressure,
 p_1 is pressure in upstream plenum,

Q is volumetric flow rate measured by nozzle(s).

Fan Curve Measurement

Locate the fan at inlet of flow bench



Fan curve:

$$\Delta p_{\text{fan}} = p_1 - p_{\text{atm}} = f(Q) \quad (5)$$

Δp_{fan} is pressure *rise* provided by fan,

p_{atm} is local ambient pressure,

p_1 is pressure in upstream plenum,

Q is volumetric flow rate measured by nozzle(s).

Note that when the DUT is a fan, $p_1 > p_{\text{atm}}$, and when the DUT is an electronic enclosure, $p_{\text{atm}} > p_1$.

Flow Bench Instrumentation

To compute flow rate, measure

- Pressure in plenum 1
- Temperature in plenum 1
- Pressure drop across the nozzle

To characterize the DUT, measure the pressure drop between ambient and plenum 1. In practice we measure

p_{atm}	with barometer
T_1	with thermocouple upstream of nozzle
$p_{\text{atm}} - p_1$	with inclined manometer or pressure transducer
$p_1 - p_2$	with manometer, pressure gage, or pressure transducer

Data Reduction (1)

Compute

$$p_1 = p_{\text{atm}} - (p_{\text{atm}} - p_1)$$
$$Q = C_d A_n Y \sqrt{\frac{2\Delta p}{\rho(1 - \beta^4)}} \quad (6)$$

- C_d is the nozzle discharge coefficient,
 A_n is the area of the nozzle throat,
 Y expansion factor to account for compressibility,
 d_t is the throat diameter,
 Δp is the measured pressure drop across the nozzle,
 ρ is the fluid density upstream of the nozzle
 $\beta = d_t/D$ is the contraction ratio,
 D is the diameter of the upstream duct.

Data Reduction (2)

The nozzles are built to ASME/ANSI specification, but are not individually calibrated. Use the generic equation for the discharge coefficient

$$C_d = 0.9986 - \frac{7.006}{\sqrt{\text{Re}_t}} + \frac{134.6}{\text{Re}_t} \quad (7)$$

$$\text{Re}_t = \frac{V_t d_t}{\nu} = \frac{4Q}{\pi d_t \nu} \quad (8)$$

- $V_t = Q/A_n$ is the velocity in the throat,
 ρ is the fluid density,
 μ is the fluid viscosity evaluated at the pressure and temperature upstream of the nozzle.

Data Reduction (3)

The analytical expression for the expansion factor is

$$Y = \left[\frac{\gamma}{\gamma - 1} \alpha^{2/\gamma} \frac{1 - \alpha^{(\gamma-1)/\gamma}}{1 - \alpha} \right]^{1/2} \left[\frac{1 - \beta^4}{1 - \beta^4 \alpha^{2/\gamma}} \right]^{1/2} \quad (9)$$

where $\gamma = c_p/c_v$ and

$$\alpha = \frac{p - \Delta p}{p}$$

$0 \leq Y \leq 1$. As $\Delta p \rightarrow 0$, $Y \rightarrow 1$.

Data Reduction (4)

An iterative procedure is required to compute Q for each measured Δp :

Initialize:

Compute and store $K_Q = A_n Y \sqrt{\frac{2\Delta p}{\rho(1 - \beta^4)}}$

Guess a value of C_d , say $C_d = 0.98$

Iterate:

1. Compute $Q = C_d K_Q$
2. Compute Re_t from equation (8)
3. Compute C_d from equation (7)
4. If the new C_d is "close enough" to the old C_d , stop.
Otherwise, return to step (1)

Data Reduction (5)

```
function Q = nozzleFlow(d,D,dp,p,T)
% nozzleFlow Volumetric flow rate of air through a long radius nozzle.

% --- Evaluate fluid properties and other constants
mu = airViscosity(T);      % kinematic viscosity
rho = p/(287*(T+273.15));  % air density from ideal gas law
bbeta = d/D;
y = expansionFactor(p,dp,bbeta,1.4);
area = 0.25*pi*d^2;
qcon = area*y*sqrt(2*dp/(rho*(1-bbeta^4))); rcon = rho*d/(area*mu);

% --- Initialize and loop until cd converges
tol = 5e-6; it = 0; maxit = 25; cdold = 0; cd = 0.9;
while abs(cdold-cd)>tol && it<maxit
    cdold = cd;
    Q = cd*qcon;
    Re = rcon*Q;
    cd = 0.9986 - 7.006/sqrt(Re) + 134.6/Re;
    it = it + 1;
end
if it>=maxit, error('No convergence after %d iterations',it); end
```

Least Squares Fit to System Curve (1)

The energy equation for the system with an unpowered fan is

$$\frac{\Delta p_{\text{sys}}}{\rho g} = h_{\text{loss}} \quad (10)$$

Recall that for pipe systems, minor losses are represented by

$$h_{\text{minor}} = K \frac{V^2}{2g}$$

where K is the so-called minor loss coefficient.

By analogy we assume that the loss through the system will also vary as the square of the average velocity.

$$h_{\text{loss}} = K \frac{V^2}{2g} = K \frac{Q^2}{2gA^2} \quad (11)$$

where A is the effective cross-sectional area of the system.

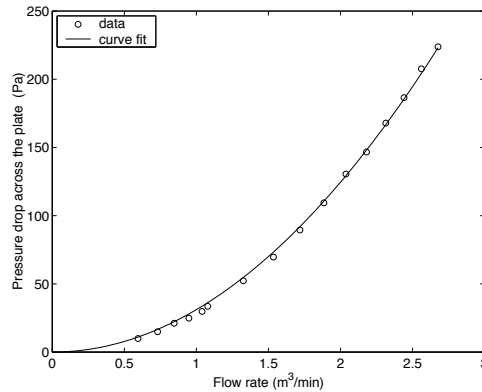
Least Squares Fit to System Curve (2)

Substitute $\Delta p_{\text{sys}} = \rho g h_{\text{loss}}$ from Equation (10) into Equation (11)

$$\Delta p_{\text{sys}} = CQ^2 \quad (12)$$

where $C = \rho K / (2A^2)$ is assumed to be a constant for the system.

Typical loss coefficient data



Least Squares Fit to System Curve (3)

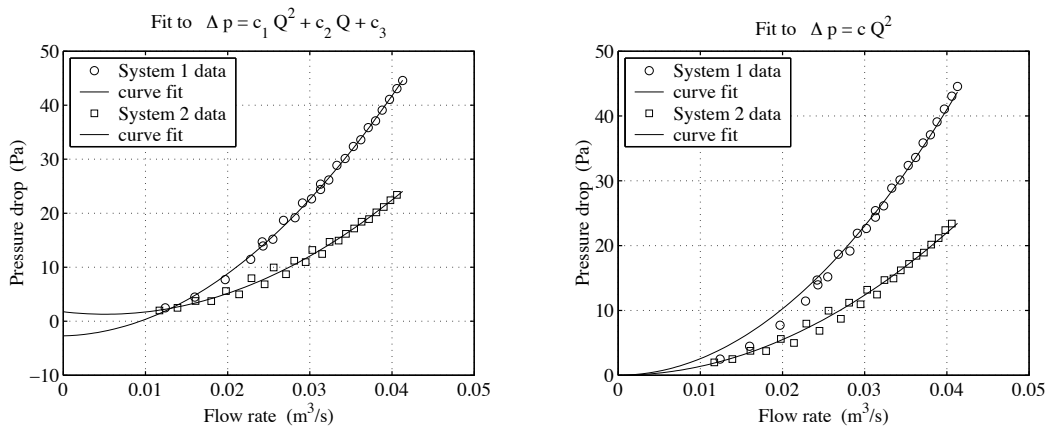
To obtain the system curve, measure a series of $(\Delta p_{\text{sys}}, Q)$ pairs. A least squares curve fit is then used to find C .

Software tools (e.g. MATLAB or spreadsheets) have built-in procedures for performing least squares fit to polynomials. Such a tool would require a curve fit of the form

$$\Delta p_{\text{sys}} = c_1 Q^2 + c_2 Q + c_3$$

But Equation (12) does not have the constant or a term linear in Q . Fortunately it is very easy to derive a simple formula that uses the least squares principle to obtain C from measured data

Least Squares Fit to System Curve (4)



Least Squares Fit to System Curve (5)

Given a set of m data pairs $(\Delta p_1, Q_1), (\Delta p_2, Q_2), \dots, (\Delta p_m, Q_m)$, write the *matrix* equation

$$\begin{bmatrix} Q_1^2 \\ Q_2^2 \\ \vdots \\ Q_m^2 \end{bmatrix} C = \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \vdots \\ \Delta p_m \end{bmatrix} \quad (13)$$

Equation (13) is an overdetermined system for the one unknown value C . Multiply both sides by $(Q_1^2, Q_2^2, \dots, Q_m^2)^T$

$$\begin{bmatrix} Q_1^2 & Q_2^2 & \dots & Q_m^2 \end{bmatrix} \begin{bmatrix} Q_1^2 \\ Q_2^2 \\ \vdots \\ Q_m^2 \end{bmatrix} C = \begin{bmatrix} Q_1^2 & Q_2^2 & \dots & Q_m^2 \end{bmatrix} \begin{bmatrix} \Delta p_1 \\ \Delta p_2 \\ \vdots \\ \Delta p_m \end{bmatrix} \quad (14)$$

Least Squares Fit to System Curve (6)

Equation (14) is the normal equation for the over determined system in Equation (13). Solving Equation (14) gives the value of C that is the least squares solution to Equation (13).

Evaluating the inner products in Equation (14) gives

$$\left(\sum_{i=1}^m Q_i^4 \right) C = \left(\sum_{i=1}^m Q_i^2 \Delta p_i \right) \quad (15)$$

This is just a scalar equation involving the one unknown value C . Solving for C gives

$$C = \frac{\sum_{i=1}^m Q_i^2 \Delta p_i}{\sum_{i=1}^m Q_i^4} \quad (16)$$

Therefore, given pairs of $(\Delta p, Q)$ data from a flow loss measurement, Equation (16) provides a simple computational formula for obtaining the C that is the least squares fit of the data to Equation (12).