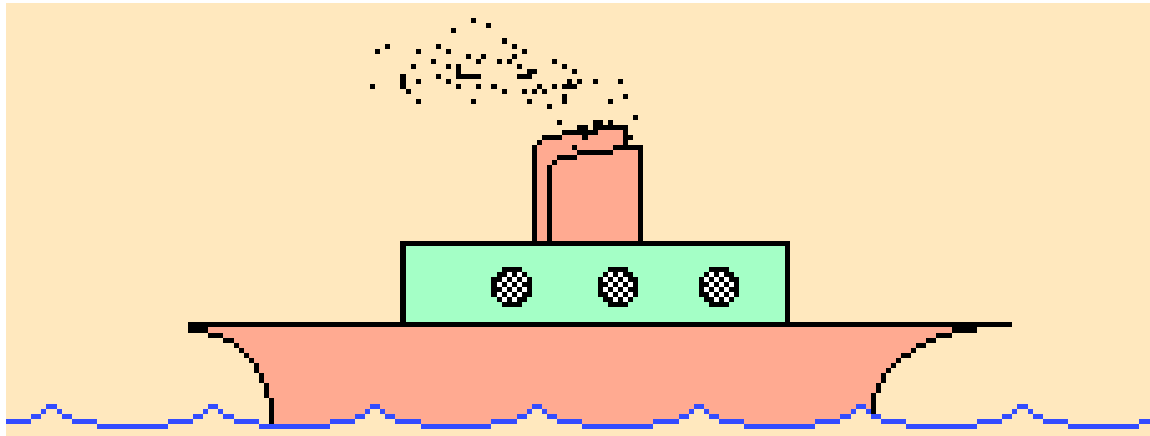


# Yacht For One Or Two



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## Overview

- Perfect Yacht
- Retrograde Analysis
- The Rest Of The Story

## The Game Of Yacht

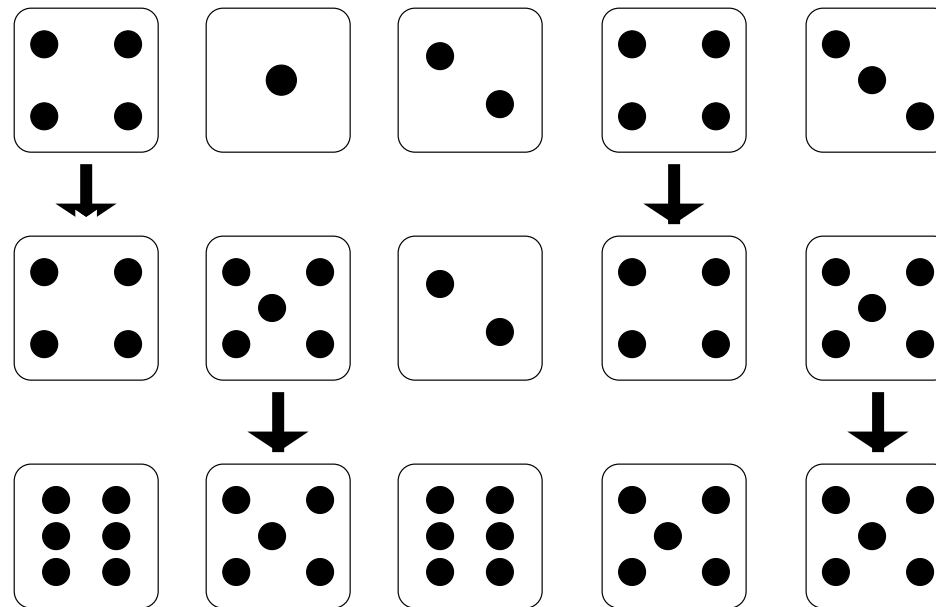
- From “Poker Dice” (1800s)
- Idea: Roll 5 dice to fill scoresheet

Lower Section		Upper Section	
Aces	<input type="text"/>	Yacht	<input type="text" value="50"/>
Deuces	<input type="text"/>	Four Of A Kind	<input type="text"/>
Threes	<input type="text"/>	Full House	<input type="text"/>
Fours	<input type="text"/>	Small Straight	<input type="text" value="30"/>
Fives	<input type="text" value="15"/>	Large Straight	<input type="text"/>
Sixes	<input type="text"/>	Choice	<input type="text"/>

## Yacht Turns

1. Roll 5 dice
2. Select subset of 5 dice to reroll
3. Select subset of 5 dice to reroll
4. Fill in score blank. If dice match no blank, “take a zero”

## A Yacht Turn



## Yacht Strategy

- Roll for most valuable blank, *but...*
- Save easy blanks for later
- Chance, Aces, Deuces blanks essentially wild
- Yacht blank is very unlikely

## Multiple-Player Yacht

- Interesting: “winner takes all”
- May be risk-centric or risk-averse
- Can play by score: still end-effects

## Yahtzee

- “History”: E.S. Lowe 1956

`http://www.hasbro.com/consumer/history/yahthist.htm`

- Slight variant
  - Slightly different blanks (13 vs. 12)
  - Subsequent Yahtzees wild
  - “Upper Section Bonus”



## Yahtzee Scoring: Upper Section Bonus

Lower Section		Upper Section	
Aces	<input type="text"/>	Yahtzee	50
Deuces	<input type="text"/>	Four Of A Kind	<input type="text"/>
Threes	<input type="text"/>	Full House	<input type="text"/>
Fours	<input type="text"/>	Small Straight	30
Fives	15	Large Straight	45
Sixes	<input type="text"/>	Three Of A Kind	<input type="text"/>
Chance	<input type="text"/>	Chance	<input type="text"/>
Bonus (63)	<input type="text"/>		

## Perfect Deterministic Games

- Deterministic 2-player game (*e.g.*, Chess): best result against perfect opponent
- Deterministic 1-player game (*e.g.*, Rubik's Cube): solve (short soln?)

## Perfect Yacht

What is “perfect” Yacht?

- Stochastic 1-player game: make moves with highest *expected value*!
- Stochastic 2-player game: make moves with highest expected winning chance
- Note: perfect information, alternating, terminating...

## Previous Work

- Tom Verhoeff, Eindhoven University of Technology  
1999: Online Optimal Single-Player Yahtzee  
`http://wwwpa.win.tue.nl/  
misc/yahtzee/`
- William Tunstall-Pedoe (date?): claimed  
`http://www.genius2000.com/  
wtpcv.html`

## Forward Analysis

- To calculate value of a pick:
  - Calculate value of each reroll
  - Weight by roll probability
  - Sum

$$E(\mathbf{pick}) = \sum_{r \in \mathbf{rerolls}} pr(r)E(r)$$

## Counting Die Rolls

dice	rolls
5	252
4	126
3	56
2	21
1	6

## Cost Of Pure Forward Analysis

- Up to 32 possible picks. Overestimate (slight)

$$(252 \cdot 32 \cdot 252 \cdot 32 \cdot 252)^{12} \approx 2 \times 10^{26}$$

- Modern machine:  $10^{11}$  insns/hour

## State Space Search

- Expected value depends only on state
- State is only scoresheet blanks plus current dice
- Can estimate value of scoresheet just before turn
- Each state:  $252^3 \cdot 32^2 \approx 2 \times 10^{10}$
- States:  $12! \approx 5 \times 10^8$
- Cost down to  $\approx 10^{19}$



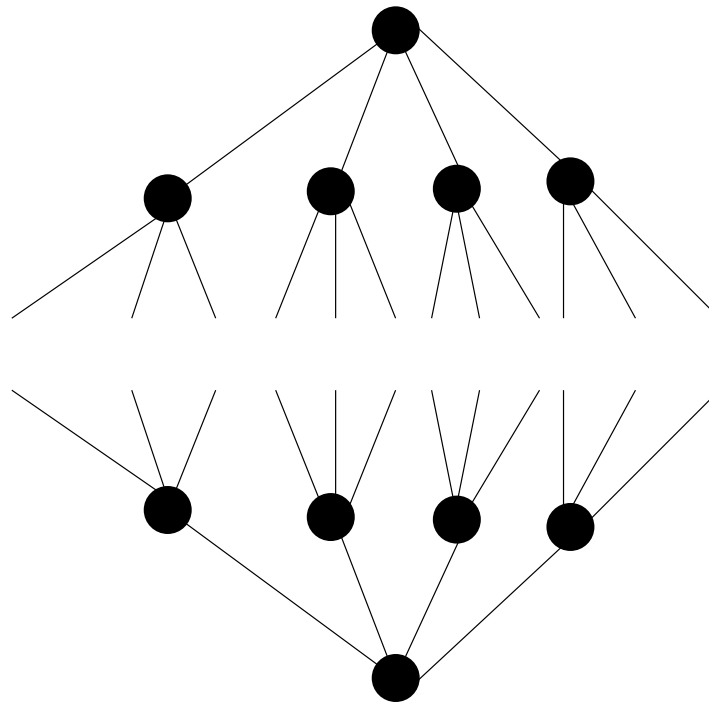
## Retrograde Analysis

- Previous slide silly
  - There are only  $2^{12}$  scoresheet states
  - Each state independent of history!
  - Start with last states

$$E(s) = \mathbf{score}(s) + E(\mathbf{succ}(s))$$

## In-Turn Retrograde Analysis

- Can use same trick for turn
- Work from end of turn to beginning



## Performance

- Current code:  $\approx 1$  sec for complete Yacht
- Yahtzee solution is harder:
  - Extra blank:  $\times 2$
  - Lower Section Bonus:  $\times 63$
  - Multiple Yahtzees: more complex code
- Current code:  $\approx 9$  min (20MB)
- (Answers? Yacht 169.8, Yahtzee 254.6)

## Yahtzee “High Score”?

- Open question (Verhoeven): Perfect solitaire play to beat previous high score?
- Can solve using retrograde analysis on target score
- Target scores in range 0..600 “interesting”

## Two-Player Yacht?

- Interesting example of “race game”
- States?

$$\sum_{k=0}^{11} \frac{1}{2} \binom{12}{k} \left[ \binom{12}{k} + 1 \right]$$

$$+ \sum_{k=0}^{10} \binom{12}{k+1} \binom{12}{k}$$

$$= 3850257$$

- Time? 12.5 days (2500 states/sec, 600 scores)

## Why?

Results show the power of

- Retrograde analysis
- Combinatorics and probabilistics
- Modern hardware

Another traditional game nears solution... Next?